A New Method of Broad-Band Equalization Applied to Microwave Amplifiers

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Abstract—**A new approach to broad-band matching which bypasses analytic gain-bandwidth theory and directly utilizes measured real frequency impedance data is applied to gain equalization and low-noise design of GaAs Schottky-barrier FET amplifiers. Neither the equalizer topology nor the analytic form of the system transfer function are initially assumed. These result from the design process. Examples include an octave-band FET amplifier design and a low-noise FET amplifier design. The equalizers are realized with lumped elements or transmission-line sections. A single basic least squares program implements the design procedure.**

I. Two-Port Equalization Problem

A NEW METHOD of broad-banding active one-ports such as passive loads and reflection amplifiers [3], and active two-ports [1] leads to the design of an equalization system to optimize gain or low-noise performance of an amplifier. In this paper we consider the equalization of microwave two-port amplifiers, particularly FET's. The method bypasses analytic gain-bandwidth theory [2] and only requires real frequency impedance data (e.g., experimental) for the given active device to be equalized. The given device is not approximated as an equivalent circuit, and neither the equalizer topology nor the analytic form of system transfer function is required. The equalization can take into account: 1) the input and output mismatch of the FET, 2) the variation of maximum available gain over the band (this level is unrealizable physically but is useful in setting performance limits), 3) the nonunilateral behavior of the FET, and 4) stability considerations.

II. Equalizer Design

A schematic diagram of the complete amplifier system is shown in Fig. 1(b). The transducer gain $T(\omega^2)$ of the system of Fig. 1(b) is given by (see Appendix A)

$$T(\omega^2)=\frac{|S_{21}(j\omega)|^2(1-|\rho_1(j\omega)|^2)}{1-|S_{11}(j\omega)|^2}\cdot\frac{(1-|\rho_2(j\omega)|^2)}{1-|S_2(j\omega)|^2}. \quad (1)$$

Here, the $S_{ij}(j\omega)$ are measured unit-normalized scattering parameters of the FET. The function $S_2(j\omega)$ is the unit-normalized back-end reflection factor of the FET with the front-end equalizer in place, as shown in Fig. 1(a), given

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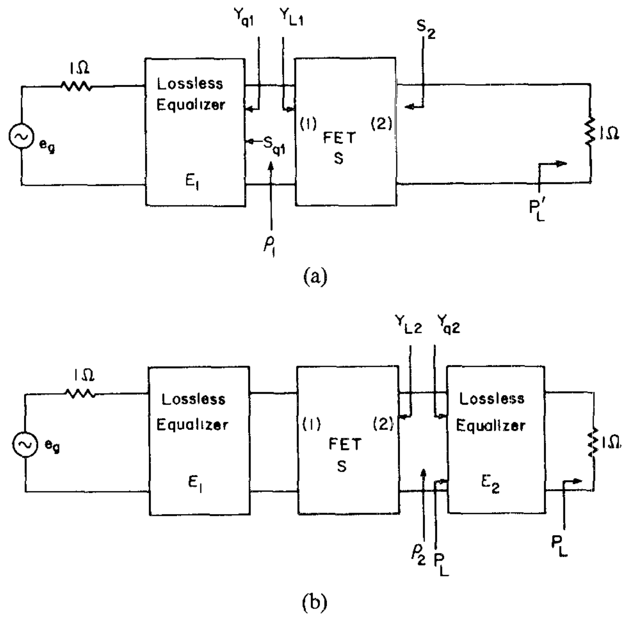

Fig. 1. Schematic of amplifier system. (a) System with input equalizer. (b) System with input and output equalizers.

by

$$S_2(j\omega)=S_{22}(j\omega)+\frac{S_{12}(j\omega)S_{21}(j\omega)S_{q1}(j\omega)}{1-S_{11}(j\omega)S_{q1}(j\omega)} \quad (2)$$

where $S_{q1}(j\omega)$ is the unit-normalized reflection factor looking in at port 2 of the front-end equalizer. The quantity $\rho_1(j\omega)$ is the *complex*-normalized reflection factor (see Appendix A, (3A)) between the admittances at port 1 of the FET, when port 2 is terminated in unit resistance, as shown in Fig. 1(a). Thus

$$1-|\rho_1(j\omega)|^2=\frac{4G_{q1}(\omega)G_{L1}(\omega)}{(G_{q1}(\omega)+G_{L1}(\omega))^2+(B_{q1}(\omega)+B_{L1}(\omega))^2} \quad (3)$$

with a similar expression in terms of impedances. The relation at port 2 of the FET has the same form but employs ρ_2, the complex-normalized reflection factor at port 2 of the FET with both equalizers in place and the appropriate admittances as shown in Fig. 1(b).

Our problem is to determine $Y_{q1}(j\omega)=G_{q1}(\omega)+jB_{q1}(\omega)$ and $Y_{q2}(j\omega)=G_{q2}(\omega)+jB_{q2}(\omega)$ to achieve a maximum flat gain over the band. These two admittances completely determine the equalizers.

At the heart of the method is the representation of an unknown conductance by a series of straight line segments [3], i.e., semi-infinite slopes, with "frequency break" or slope discontinuity points at $0<\omega_1<\omega_2\cdots<\omega_n$. There is a total of r_k mho of conductance change for the straight line segment of G_q between ω_{k-1} and ω_k. The r_k are the unknowns which as shown below specify the equalizer. The ω_k break points can be chosen to divide the band evenly or be distributed unevenly based on the behavior of the real frequency load data. The choice of ω_n beyond which the conductance and gain are zero depends upon the rolloff (and hence equalizer complexity) desired.[1] Bringing this final break point closer to the passband edge results in higher order equalizers approaching the theoretical optimum gain-bandwidth limit.

Having chosen the frequency break points for the straight line representation, the equalizer conductance at any frequency is given as a linear combination of the unknown individual conductance excursions r_k of each of the line segments

$$G_q(\omega)=r_0+\sum_{k=1}^{n}a_k(\omega)r_k=r_0+\boldsymbol{a}^T(\omega)\boldsymbol{r} \tag{4}$$

where r_0 is the dc conductance and $\boldsymbol{r}^T=(r_1r_2\cdots r_n)$. That is, $\boldsymbol{r}$ is a column vector and $\boldsymbol{r}^T$ is its transpose, a row vector. The components $a_k(\omega)$ of the vector $\boldsymbol{a}(\omega)$ are known in terms of the break points since G_q is just a connected sequence of straight line segments.

$$a_k(\omega)=\begin{cases}1, & \omega_k\leqslant\omega\\ \dfrac{\omega-\omega_{k-1}}{\omega_k-\omega_{k-1}}, & \omega_{k-1}\leqslant\omega\leqslant\omega_k.\\ 0, & \omega<\omega_{k-1}\end{cases} \tag{5}$$

The advantage of this representation is that the minimum susceptance $B_q(\omega)$ corresponding to $G_q(\omega)$ is also a linear combination of the same unknown conductance excursions

$$B_q(\omega)=\boldsymbol{b}^T(\omega)\boldsymbol{r}. \tag{6}$$

The components $b_k(\omega)$ of the vector $\boldsymbol{b}(\omega)$ are given in terms of the break points

$$b_k(\omega)=\frac{1}{(\omega_k-\omega_{k-1})\pi}\int_{\omega_{k-1}}^{\omega_k}\ln\left|\frac{y+\omega}{y-\omega}\right|dy. \tag{7}$$

This integral has a simple closed-form evaluation [4].

Note that alternately the actual values of $G_q(\omega)$ at the frequency break points may be used as the unknowns. The defining equations for G_q and B_q remain linear, and the new coefficients are simple linear combinations of the a_k and b_k.

Once the line segments describing $G_q(\omega)$ have been determined, an appropriate rational approximation $\hat{G}_q(\omega)\approx G_q(\omega)$ can be found by any convenient means (such as approximating the line segments by a rational function determined by least squares). The functional form of $\hat{G}_q(\omega)$ will determine equalizer network topology, and $\hat{Y}_q(j\omega)$ as obtained [4] from $\hat{G}_q(\omega)$ can be realized as a Darlington reactance two-port (the equalizer) with a resistive termination (the generator impedance).

The design process is implemented by first determining the ideal straight line segment solution for $G_{q1}(\omega)$ by equalizing the first factor in (1) to a maximum flat gain level. The approximation $\hat{G}_{q1}(\omega)$ to this straight line solution is then determined and then $\hat{Y}_{q1}(j\omega)$. $S_2(j\omega)$ is calculated (2) with the front-end equalizer admittance $\hat{Y}_{q1}(\omega)$ present. With the first factor known, a straight line segment solution for $G_{q2}(\omega)$ is now determined by equalizing the second factor of (1) to a maximum overall flat gain level. An approximation $\hat{G}_{q2}(\omega)$ to this line solution followed by $\hat{Y}_{q2}(j\omega)$ defines the back-end equalizer and completes the design process. As a variant of the above procedure, the equalization of $|S_{21}(j\omega)|^2$ may be distributed in any desired fashion between the first and second factors of (1). This corresponds to compensating for amplifier taper in either the input or output equalizers or a combination thereof. For example, we may choose to equalize gain variations of $|S_{21}(j\omega)|$ *only* in the first factor. Then the second factor only provides impedance matching. In this case the amplifier output is matched to the line, and gain taper is compensated by the input equalizer, or we may choose to match the input and compensate for gain taper in the output equalizer.

It should be pointed out that the function (1) exactly specifies the gain with the two equalizers in place. However, the design procedure can be iterated one or more times in order to improve the maximum flat gain level. If two successive iterations result in negligible change, the procedure is terminated. In general, if $|S_{12}|\ll 1$ (i.e., amplifier approximately unilateral), no iteration is necessary. In this case, (2) indicates that $S_2(j\omega)=S_{22}(j\omega)$.

Note that the numerical operations involved in determining the straight line solution are well conditioned. This is because: 1) the representation of $G_q(\omega)$ and $B_q(\omega)$ are expressed as *linear combinations* of the real unknowns, r_k; 2) the gain is at most quadratically dependent on the unknowns because of the form of the gain equation; and 3) the effect of line segments remote from the frequency in question on $B_q(\omega)$ is small. In essence, the straight line approximation of the equalizer conductance or resistance provides an idealized solution that can be obtained with a single basic least squares numerical method applicable to all loads and equalizers (see Appendix B). In many cases, an interactive procedure utilizing APL or a programmable calculator is sufficient to implement the computations.

Typically, the element values resulting from the synthesis of the approximation to the line segment solution are

[1] A useful measure of complexity (segment number of equalizer elements if all zeros of transmission are at infinity) is one-half the degree difference between numerator and denominator of the rational function approximating the line segment characteristic. This number m can be estimated by assuming stopband conductance rolloff proportional to ω^{-2m} and choosing some realistic figure, say between 10 and 20 dB, for the attenuation of conductance between passband edge frequency ω_c and "zero conductance point" ω_n. Thus, if we accept 13 dB down for the conductance at $\omega_n=1.25\ \omega_c$, then $2m\log_{10}1.25\cong 13/10$ and $m\cong 7$.

practically realizable. This result arises from the flexibility of the technique. We are not constrained to using a particular form of transfer function or a particular network topology. If, as a part of the approximation process coefficients that are too large or coefficients of similar magnitude but opposite sign result, we can discard this approximation and readjust the coefficients by using an approximation of a higher order of complexity. In this manner, we can settle upon network element values as well as a network topology of minimum complexity which realizes the design goals.

III. Example IA—Octave-Band Gain Equalization of a GaAs Schottky-Barrier FET Amplifier—Minimum Susceptance Case

In order to demonstrate the technique, consider gain equalization of a Schottky-barrier FET across an octave band. The straight line conductance solutions are obtained by using a linear least square routine to minimize the in-band deviation from a chosen gain level. That is, we minimize

$$E=\sum_{j=1}^{m}\left[\frac{T(\omega_j^2)}{T_0(\omega_j)}-1\right]^2 \tag{8}$$

with respect to the unknown conductance excursions r_k. Here the objective function $T(\omega^2)$ is given by (1), and $T_0(\omega_j)$ is the chosen gain level at the frequency ω_j. Choosing the level $T_0(\omega_j)$ to vary with frequency is equivalent to introducing a known gain weighting function and permits compensation for amplifier taper, which is included in the factor $S_{21}(j\omega)|^2$ in (1). Because of the form of the gain (1) and the representation of $G_q(\omega)$ and $B_q(\omega)$ as linear combinations of the unknowns, a linear least square routine can be set up with a Jacobian which is simply expressed in closed form. By employing more sophisticated numerical techniques, such as constrained least squares or nonlinear programming, constraints such as stability requirements can be included in the technique. (See Appendix B for a short discussion of the basic procedure.)

The numerically specified scattering parameters of the NEC 1-μm gate GaAs Schottky-barrier FET [5] as function of frequency were used for this example. The straight line segment conductance solution and the rational approximation for the front-end equalizer are shown in Fig. 2(a). The straight line segment conductance solution and the rational approximation for the back-end equalizer are shown in Fig. 2(b). In both cases, the straight line conductance solution was approximated by $\hat{G}_q(\omega)=[P_5(\omega^2)]^{-1}$ where $P_5(\omega^2)$ is an appropriate even polynomial of degree 10. The resulting equalizer networks are five-element lowpass LC ladders, whose element values were determined using a Cauer continued-fraction expansion of $\hat{Y}_q(j\omega)$, obtained from $\hat{G}_q(\omega)$.

No iteration of the procedure was necessary in this case because $|S_{12}|$ was sufficiently small. This equalized amplifier system is shown in Fig. 3. Note that seven slopes suffice to describe these equalizers over a band of dc to 10 GHz. The equalized gain response of the FET amplifier system was 8.25 $\pm$0.58 dB across the octave band of 4.0–8.0 GHz and is shown on Fig. 4 as dotted lines. The maximum available gain of this transistor is also given for comparison.

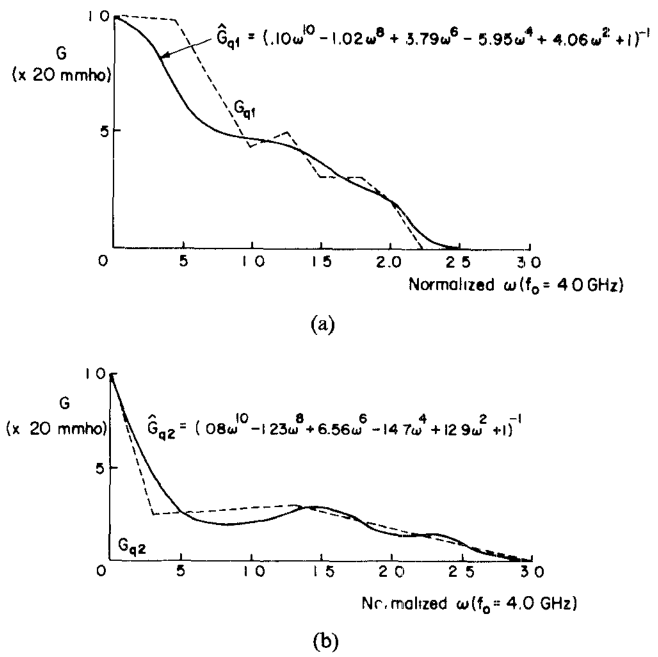

Fig. 2. (a) Front-equalizer conductance designed for NEC 1-μm gate GaAs Schottky-barrier FET. (b) Back-equalizer conductance for this FET: G_{q1}, G_{q2}—Line segment equalizer conductance solution; $\hat{G}_{q1}$, $\hat{G}_{q2}$—Rational approximations to G_{q1}, G_{q2}.

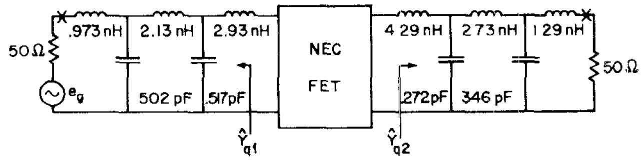

Fig. 3. Compensated FET amplifier based on Fig. 2 and with low-pass equalizers, (X denotes bias decoupling capacitor).

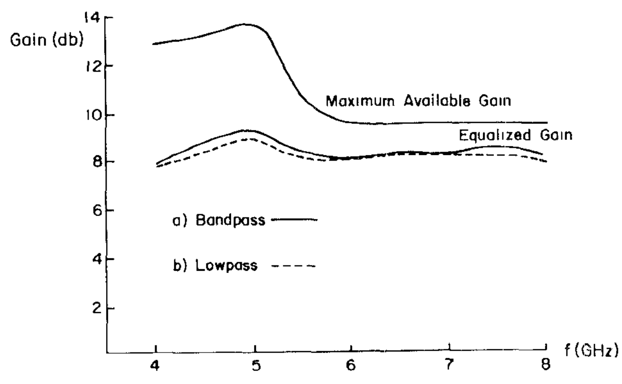

Fig. 4. Gain response of equalized FET amplifier using a) bandpass equalizers (8.53$\pm$0.72 dB), (with shunt L's), and b) lowpass equalizers (8.25$\pm$0.58 dB), (based on Fig. 2).

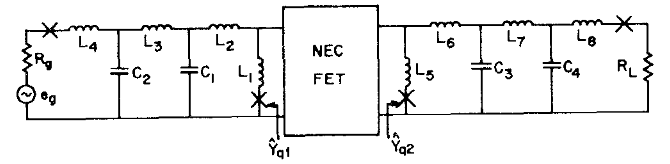

Fig. 5. Equalized FET amplifier using shunting inductors for zero dc gain, (X denotes bias decoupling capacitor): $R_g = R_L = 50\ \Omega$; $L_1 = 6.54$ nH, $L_2 = 4.25$ nH, $L_3 = 3.48$ nH, $L_4 = 1.58$ nH, $C_1 = 0.3456$ pF, $C_2 = 0.4309$ pF; $L_5 = 17.75$ nH, $L_6 = 7.27$ nH, $L_7 = 5.60$ nH, $L_8 = 1.32$ nH, $C_3 = 0.1295$ pF, $C_4 = 0.3084$ pF (see Fig. 4a)).

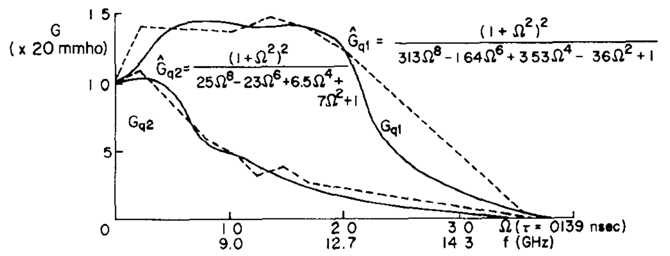

Fig. 6. Equalizer conductances for noise figure equalization: G_{q1}, G_{q2}—front and back line segment equalizer conductances; $\hat{G}_{q1}$, $\hat{G}_{q2}$—rational approximations to G_{q1}, G_{q2} with $\Omega = \tan \omega\tau$.

IV. Example IB—Nonminimum Susceptance Equalizer

The previous discussion has dealt with $Y_q(\omega)$ as minimum susceptance (i.e., no poles of susceptance at any real frequency, so that $G_q(\omega)$ and $B_q(\omega)$ are uniquely related by Hilbert transforms) and results in series inductances at the FET ports of the equalizers. Permitting a shunt inductance at the FET port of the equalizer [6] results in a bandpass gain response with a dc gain of zero, but the equalizer admittance $Y_q(j\omega)$ is now nonminimum susceptance since it has a pole at dc. We handle this case by resonating the shunt inductance L to the load at the lower passband edge and then taking this circuit component into account by including it as part of the load data (i.e., the revised load susceptance is $B_L(\omega) - 1/(\omega L)$). The remaining equalizer is then minimum susceptance, and the original optimization procedure may be employed. Note that if further refinement were required, the shunt L could be optionally trimmed after the approximation to the minimum susceptance equalizer was completed.[2]

By using a shunt inductance at the FET ports of the equalizers followed by two five-element low-pass LC-ladder networks (as shown in Fig. 5) a bandpass gain response of 8.53 ± 0.72 dB results across the octave band of 4.0–8.0 GHz (shown on Fig. 4 as a solid line). Again, no iteration was necessary in this case since $|S_{12}|$ was sufficiently small. In comparison to the low-pass solution with a minimum gain of 7.66 dB, the bandpass solution shows an improved minimum gain of 7.81 dB.

V. Example II—Gain Equalization and Low-Noise Design of a GaAs Schottky-Barrier FET Amplifier Using Transmission-Line Equalizers

The technique described here is not limited to gain equalization as an objective function to be optimized while satisfying gain-bandwidth restrictions. The objective function may be gain, noise figure, noise measure, or any other function that can be formulated in terms of impedances or admittances.

The noise figure of an amplifier is dependent upon the Thévenin impedance of an equalizer as seen from the input of the amplifier and is given by [7]

$$F(\omega) = F_{\min}(\omega) + \frac{R_f}{G_{q1}(\omega)}\left[\left(G_{q1}(\omega) - G_f(\omega)\right)^2 + \left(B_{q1}(\omega) - B_f(\omega)\right)^2\right] \quad (9)$$

where $F_{\min}(\omega)$ is the minimum noise figure of the amplifier, $Y_f(j\omega) = G_f(\omega) + jB_f(\omega)$ is the source admittance producing $F_{\min}(\omega)$, and R_f is a parameter. All of these are measured properties of a particular FET. Consequently, we can use (9) to determine $Y_{q1}(j\omega)$ (the front-end equalizer) and employ the second factor of (1) to determine $Y_{q2}(j\omega)$ (the back-end equalizer) for gain compensation [8]. Note that, in general, the optimum source admittance for noise figure is different from the source admittance for maximum available gain. As a result, the input equalizer designed for noise performance will reduce the transducer gain of the transistor.

For this example, the equalizing networks were designed by employing a cascade of commensurate transmission lines and stubs of length L (delay $= \tau = L/c$, $c = 3 \times 10^8$ m/s) rather than lumped elements as in the previous example. To accomplish this, the modified Richards' transformation [9]

$$\Omega = \tan(\omega\tau) \quad (10)$$

was employed as the frequency variable in the design process. In the cascade connection, when this frequency transformation is employed, driving-point network functions are rational in the Ω domain.

The scattering and noise parameters of the HP 1-μm gate GaAs Schottky-barrier FET [10] were used for this example. The delay length τ is chosen to give commensurate lines that are $\lambda/4$ in length at 1.5 times the high-frequency limit of the passband.[3] The straight line conductance solutions were obtained in the Ω domain (Fig. 6). They were then approximated by

$$\hat{G}_q(\Omega) = \frac{(1+\Omega^2)^n}{P_m(\Omega^2)} \quad (11)$$

where $P_m(\Omega^2)$ is an even strictly positive polynomial of

[2]For this case we could also proceed by writing $B_q(\omega) = b^T r + (-1/\omega)1/L$ and treat $1/L$ as an additional unknown.

[3]We cannot impose bandpass symmetry on the gain characteristic by centering the $\lambda/4$ point in the band; instead it must be chosen at some convenient point above the high-frequency edge of the passband. This defines the useful band which is periodically repeated for the equalizer.

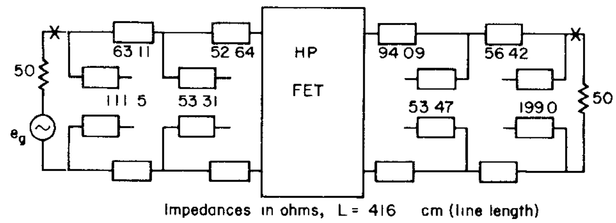

Fig. 7. Noise figure compensated FET amplifier configuration designed from $\hat{G}_{q1}$, $\hat{G}_{q2}$ of Fig. 6 (X denotes bias decoupling capacitor).

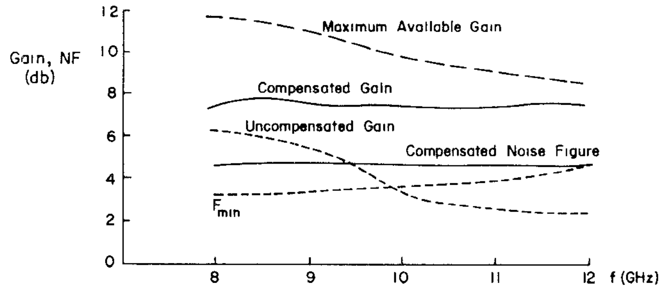

Fig. 8. Noise figure and gain of FET amplifier designed from $\hat{G}_{q1}$, $\hat{G}_{q2}$ of Fig. 6.

degree $2m \geqslant 2n$. Equation (11) is the prescribed form of a rational conductance function in order to be realized by a cascade of n commensurate transmission lines and $m-n$ commensurate stubs [9]. Both front and back equalizers employ two transmission lines and two open-circuited stubs as shown in Fig. 7. For this example, the delay length τ was chosen to make the commensurate lines $\lambda/4$ in length at 18.0 GHz. Characteristic impedances were determined by using a Richards' extraction algorithm with remainder truncation [11]. Fig. 8 shows the noise figure and gain of the compensated amplifier. The amplifier noise figure is 4.50 ± 0.05 dB with an associated gain of 7.13 ± 0.32 dB across the 8.0–12.0-GHz band.

VI. Conclusions

The real frequency technique described in this paper is straightforward, yet it can be applied to a broad range of practical examples. It handles complicated loads which are too difficult for analytic procedures and may be preferable even when analytic methods can be employed. Furthermore, the technique processes measured real frequency data without assuming analytic models, system transfer function, or equalizer topology. The programming can include constraints such as stability requirements. Also, a single least squares program can be used for all gain equalization problems. Finally, the technique permits rapid convergence to a realizable physical design.

Appendix A
Derivation of Equalized Two-Port Gain Equation

As given in the text (1) is a special form of the transducer gain equation for the equalized FET amplifier which permits direct application of the design method discussed in the paper. Thus this equation only involves the measured S parameters (unit normalization) of the FET, the port impedances of the FET (computable from the S parameters) and the unknown port impedances of the front and back equalizers seen from ports 1 and 2 of the FET. These latter functions determine the equalizers. The derivation is as follows. Consider Fig. 1(a) and suppose the incident voltage (unit normalization) at port 1 of the FET to be a_1. Then, the power transmitted across port 1 of the FET is

$$P_0 = |a_1|^2(1-|S_{11}|^2) \tag{1A}$$

and the power to the 1-Ω load terminating the FET in Fig. 1(a) is

$$P_L' = |a_1|^2|S_{21}^2| = \frac{P_0|S_{21}|^2}{1-|S_{11}|^2} \tag{2A}$$

where the S_{jk} are the unit-normalized S parameters of the FET. But with ρ_{1_2} the reflection factor at port 1 of the FET, complex-normalized to Y_{q1}

$$|\rho_1|^2 = \left|\frac{Y_{q1}^* - Y_{L1}}{Y_{q1}+Y_{L1}}\right|^2 \tag{3A}$$

the power P_0 delivered to the FET is

$$P_0 = P_A(1-|\rho_1|^2) \tag{4A}$$

where P_A is the available power from the 1-Ω generator e_g. Thus for Fig. 1(a),

$$T'(\omega^2) = \frac{P_L'}{P_A} = \frac{(1-|\rho_1|^2)|S_{21}|^2}{1-|S_{11}|^2}. \tag{5A}$$

Now refer to Fig. 1(b) and port 2 of the FET. Suppose a' to be the voltage wave variable incident on E_2 with complex normalization to the FET admittance Y_{L2}. The power delivered to the FET port 2 load is then

$$P_2 = |a'|^2(1-|\rho|^2)$$

where ρ is the load reflection factor complex-normalized to the FET admittance Y_{L2}. a' is of course invariant to the termination when the system to the left of port 2 is fixed. Here ρ has no subscript since this equation is for any load.

Thus, if port 2 is terminated in 1 Ω (as in Fig. 1(a)), the power P_L' of (2A) is transmitted, and, in this case

$$|\rho|^2 = \left|\frac{Y_{L2}^*-1}{Y_{L2}+1}\right|^2 = \left|\frac{1-Y_{L2}}{1+Y_{L2}}\right|^2 = |S_2|^2 \tag{6A}$$

where S_2 is just the 1-Ω normalized port 2 reflection factor of the FET with equalizer E_1 in place (2). Then the transmitted power is

$$P_2 = P_L' = |a'|^2(1-|S_2|^2). \tag{7A}$$

Finally, if the equalizer E_2 terminates the FET, we now have the power delivered at port 2 of the FET and $\rho \equiv \rho_2$. In this case

$$P_2 = |a'|^2(1-|\rho_2|^2) \tag{8A}$$

where

$$|\rho_2|^2 = \left|\frac{Y_{L2}^* - Y_{q2}}{Y_{L2}+Y_{q2}}\right|^2. \tag{9A}$$

From (7A) we find $|a'|^2$, so that using P_L' from (2A) and (4A) and substituting in (8A) we find P_2 with both equalizers in place. Since E_2 is lossless, this is the same as P_L, the power to the final 1-Ω termination of E_2.

Thus

$$P_L' = \frac{|S_{21}|^2(1-|\rho_1|^2)}{1-|S_{11}|^2} P_A$$

and from (8A)

$$P_L = P_2 = \frac{(1-|\rho_2|^2)P_L'}{(1-|S_2|^2)}$$

so that finally we have (1) of the text:

$$\frac{P_L}{P_A} = T(\omega^2) = \frac{(1-|\rho_1|^2)(1-|\rho_2|^2)|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_2|^2)}. \qquad (10A)$$

As mentioned in the text, if the FET is unilateral, ($S_{12}=0$) $S_2 = S_{22}$ in (10A).

Appendix B
Least Squares Matching

We show here how a simple Gauss–Newton least squares routine can be applied to the matching problem. In particular, it will be clear that a single program can be used for all such problems independent of the complexity of the load or equalizer.

The fundamental gain function to be optimized is generally of the form of (3) in the main text and may be applied to reflection amplifiers or passive loads [3], or two-ports as in (1). Thus for the discussion below let

$$\text{Gain} = t(\omega) = \frac{4G_q(\omega)G_L(\omega)}{(G_q(\omega)+G_L(\omega))^2 + (B_q(\omega)+B_L(\omega))^2}. \qquad (1B)$$

Then define an error function as the fractional deviation of $t(\omega)$ from some specified gain function $g_0(\omega)$, which of course may be constant.

$$e(\boldsymbol{r},\omega) = \frac{t(\boldsymbol{r},\omega)-g_0(\omega)}{g_0(\omega)} = g(\boldsymbol{r},\omega)-1, g(\boldsymbol{r},\omega) = \frac{t(\boldsymbol{r},\omega)}{g_0(\omega)}. \qquad (2B)$$

As in (4) and (6)

$$G_q(\boldsymbol{r},\omega) = r_0 + \boldsymbol{a}^T\boldsymbol{r} \qquad (3B)$$

$$B_q(\boldsymbol{r},\omega) = \boldsymbol{b}^T\boldsymbol{r}. \qquad (4B)$$

The function to be minimized over the passband by appropriate choice of $\boldsymbol{r}$ is $E=\sum_{\omega_j} e^2(\boldsymbol{r},\omega)$. The ω_j are sampling points for the error in the passband (we have used of the order of 20 such points). Note that these sampling points are *not* the ω_k break points which define $G_q(\omega)$. The latter are relatively few in number.

Let $\boldsymbol{r}_0$ be an initial guess for $\boldsymbol{r}$. For example $\boldsymbol{r}_0$ can be chosen by assuming a conductance match to the load at the frequency break points ω_k. For some initial choice $\boldsymbol{r}_0$,

$$G_{q0}(\omega) \equiv G_q(\boldsymbol{r}_0,\omega) \quad B_{q0}(\omega) \equiv B_q(\boldsymbol{r}_0,\omega) \quad e_0(\omega) \equiv e(\boldsymbol{r}_0\omega) \qquad (5B)$$

and

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{\delta}, \quad e(\boldsymbol{r},\omega) = e_0(\omega) + \boldsymbol{f}^T(\omega)\boldsymbol{\delta}. \qquad (6B)$$

In (6B) the unknown increments with respect to the initial guess $\boldsymbol{r}_0$ are the components of the column vector $\boldsymbol{\delta} = (\delta_1, \delta_2, \cdots, \delta_n)^T$, and $\boldsymbol{f}(\omega)$ is the gradient at $\boldsymbol{r}=\boldsymbol{r}_0$

$$\boldsymbol{f}(\omega) = \frac{\partial e(\boldsymbol{r},\omega)}{\partial \boldsymbol{r}_0} = \frac{\partial g(\boldsymbol{r},\omega)}{\partial \boldsymbol{r}_0}. \qquad (7B)$$

The gradient vector $\boldsymbol{f}(\omega)$ is simply expressed in *explicit algebraic* form for all gain problems by (1B) and (7B). Thus

$$\boldsymbol{f}(\omega) = \frac{\partial g(\omega)}{\partial G_{q0}} \boldsymbol{a}(\omega) + \frac{\partial g(\omega)}{\partial B_{q0}} \boldsymbol{b}(\omega). \qquad (8B)$$

We now obtain a set of linear algebraic equations in the unknowns δ_k by setting

$$\frac{\partial}{\partial \boldsymbol{\delta}}\left(\sum_{\omega_j} e^2(\boldsymbol{r},\omega)\right) = 0. \qquad (9B)$$

The equations obtained by substituting (6B) into (9B) are

$$\left[\sum_{\omega_j} \boldsymbol{f}(\omega)\boldsymbol{f}^T(\omega)\right]\boldsymbol{\delta} = -\sum_{\omega_j} e_0(\omega)\boldsymbol{f}(\omega). \qquad (10B)$$

The quantity $\sum_{\omega_j}\boldsymbol{f}\boldsymbol{f}^T$ is a sum of dyads, hence an $n \times n$ matrix. It should be clear that (9B) and its solution for $\boldsymbol{\delta}$ can be programmed (as well as additional iterations) into a standard form for all loads. We require as input the given load data at the ω_j, the initial guess $\boldsymbol{r}_0$ at the frequency break points ω_k, and the chosen gain values $g_0(\omega_j)$. The method described is a Gauss–Newton procedure but the basic ideas remain the same if more refined methods such as the Levenberg–Marquardt technique [12] are used.

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A Generalized Multiplexer Theory

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***Abstract*—A general direct analytical design process is presented for multiplexers having any number of channels with arbitrary channel complexity, bandwidths, and interchannel spacings. The theory assumes initially that independent doubly terminated designs are available for the individual filters, and formulas for modifications to parameters associated with the first two resonators are developed to match the multiplexer. These formulas are approximate, and the limitations of the theory are indicated with several computed examples. The theory is applied to the design of a five-channel interdigital multiplexer.**

A first-stage immittance compensation scheme is described which improves the design for limiting cases, but the theory of complete immittance compensation which handles even contiguous channel operation is reserved for a companion paper.

I. Introduction

IN TWO previous papers, direct design formulas were presented for bandpass channel diplexers [1], [2]. In this and a companion paper [3] the procedure is extended to the general multiplexer case having any number of channels, arbitrary channel complexity, and arbitrary channel bandwidths and center frequency allocations.

The theory may be developed in two distinct phases. In the first phase, to which this paper is devoted, design formulas are derived for interacting channel filters having direct connection (all in series or all in parallel) without additional immittance compensation networks. This is an important practical configuration and gives acceptable results for a wide variety of common specifications, as demonstrated by computer analysis and by experimental results presented. The main limitation is that the channels may not be spaced too closely in frequency.

In the second phase of the theory, consideration is given to the design of immittance compensation networks. Although a number of possible schemes for immittance compensation are feasible, it has been found possible to design multiplexers on a manifold of uniform impedance where the phase shifts between the various filters on the manifold not only serve to separate the filters physically, but also act as immittance compensation networks. The results are expressed in the form of closed formulas, and little or no computer optimization is required. This extended theory may be applied even to the limiting case of contiguous band coverage and is the subject of the companion paper [3].

Initial consideration has been given to the possibility of designing multiplexers on the basis of exact synthesis, but it has become apparent that this is possible only for certain restrictive classes of networks. For example, in the diplexer case, if two networks have input impedances Z and $1-Z$ and are connected in series to a resistive generator of 1-Ω internal impedance, then there is a perfect match at all frequencies at the input port. Power is distributed to the two networks as a function of frequency according to the frequency variation of Re Z and $1-$Re Z. If there is perfect transmission in one channel at the set of frequencies $\omega=\omega_i$, then there must be infinite attenuation in the other channel at $\omega=\omega_i$. Assuming that the set of ω_i are chosen such that there is equiripple transmission in the passbands, then, except for one very special case[1], the stopbands will not possess an equiripple behavior. In this example, there is no frequency region where both channels possess a common stopband. If they do, then the return loss at the common port will be finite except at a finite number of frequencies. This response may be made exactly equiripple in an optimum manner over the two individual passbands. However, the reflection at the individual channel outputs will not, in general, be equiripple. The only possible case in which this can be true is when

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[1]This occurs when the minimum return loss level in the passband is approximately equal to the minimum insertion loss level in the stopband.

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